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Stability of Satellite Orbits around Asteroid 243 Ida

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Abstract

Research into the stability of gravitational orbits in rotating, non-spherical potentials has both theoretical merit and practical applications for future visits to irregular asteroids by spacecraft. The current study numerically integrates orbits in a model potential for asteroid 243 Ida over a wide range of orbital parameters. Regions of stability are studied and some very interesting phenomena are observed. Included in this article are the observations of a fractal-like structure to stability-instability boundaries in parameter space and a process of orbital self-preservation in the case of slow asteroid rotation.

Introduction

There has been significant work in recent years aimed at studying the orbits of satellites in rotating, non-spherical potentials. An obvious real world realization of such potentials are those around irregular asteroids. Orbits in such potentials will generally be far from Keplerian; gravitational perturbations can cause significant changes in satellite energy and angular momentum which may lead to either a crash or an escape trajectory. This then brings up the critical question of orbital stability.

In this article we numerically study satellite orbits around asteroid 243 Ida (Belton *et al.* 1996) and examine some interesting qualitative results. The Ida asteroid is highly elongated, with approximate dimensions of (60 x 25 x 18) km. A detailed shape model was produced by Thomas *et al.* (1996). The mass of Ida is taken here to be $M = 4.2 \times 16^{10}$ kg (Petit *et al.* 1997). We used a 44-sphere model of Ida in order to model its potential, courtesy of Jean-Marc Petit. This model consists of 44 spheres with different masses and positions which together replicate Ida's potential extremely well. Our coordinate system is such that the centre of mass is located at the origin, the x-axis is aligned with the long axis, the y-axis with the middle, and the z-axis with the short axis. There is reason to believe (Burns and Safronov 1973) that asteroids will tend to spin in a state of principle axis rotation, meaning that its rotational axis will be aligned with the symmetry axis about which the moment of inertia is maximized (in this case, the z-axis). Ida has in fact been observed to be in a state of principle axis rotation (Belton *et al.* 1996). All results presented here are based on this type of asteroid rotation.

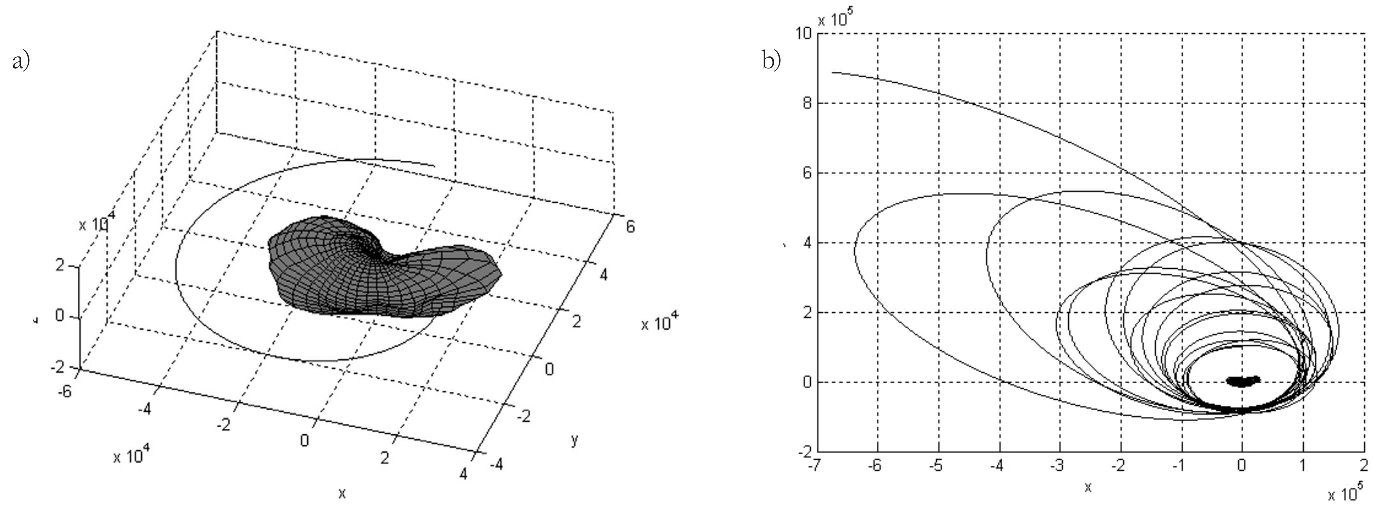

Figure 1 An unstable orbit is one which results in either a crash into Ida or an escape trajectory. (a) Shows a crash trajectory. (b) Shows an escape trajectory.

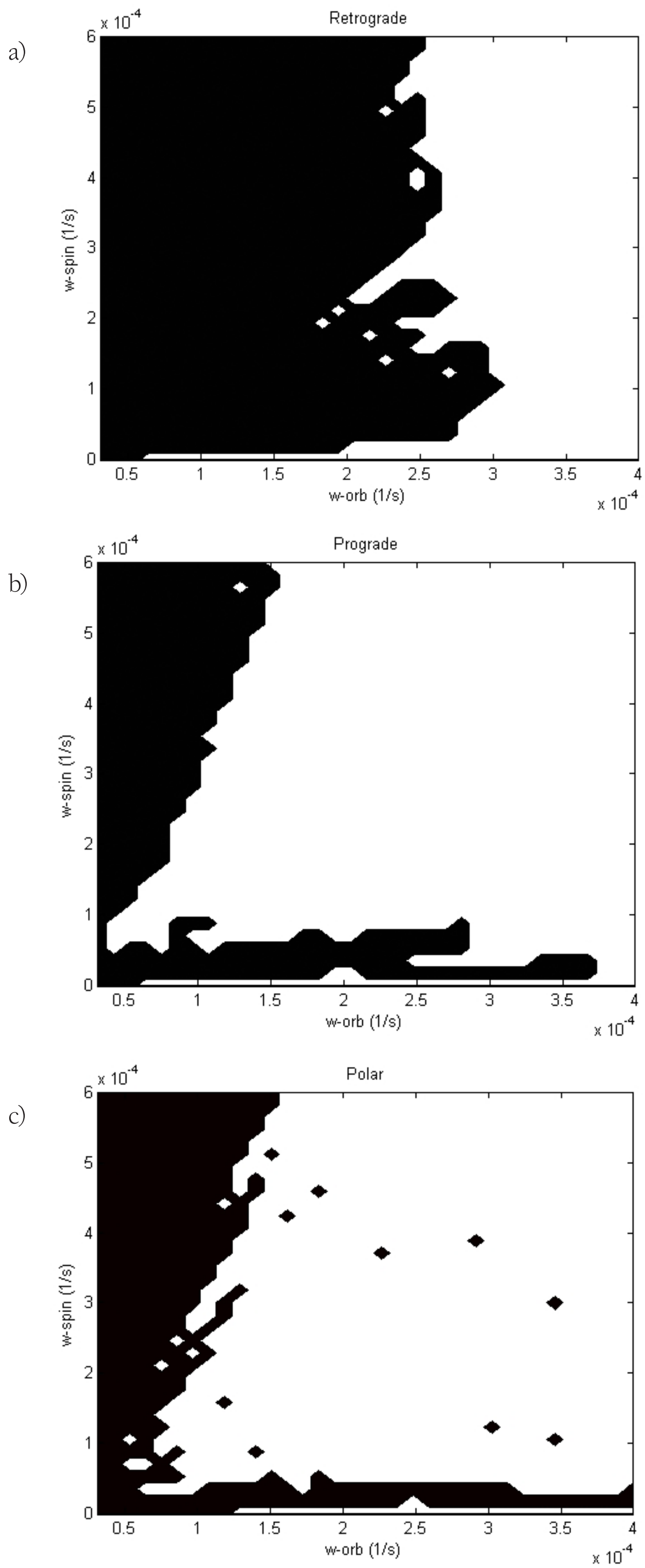

Figure 2 The orbital stability (over a 1000-year simulation) for varying values of ω_{spin} and ω_{orb} . Black represents a stable orbit while white represents an unstable one. Each figure has a 35x35 resolution. (a) Stability plot for retrograde orbits. (b) Prograde. (c) Polar.

Stability

Using the 44-sphere model, a large number of satellite orbits were numerically integrated over a range of orbital parameters. The high performance computing network SHARCNET was used to integrate these orbits over a 1000 year period; this corresponds to about 160 thousand to 2 million orbital periods, depending on the distance from the satellite to Ida. If the satellite crashed or escaped, its orbit was deemed to be unstable. Figure (1,a) and (b) show crash and escape trajectories respectively (note that Ida is displayed in its final position).

There are two basic parameters over which stability was tested. First is the angular frequency of the asteroid rotation, ω_{spin}. The second parameter is the angular frequency of the satellite orbit, ω_{orb}. In this study, we define this frequency such that

$$\omega_{orb} = \sqrt{\frac{G\ M}{a_0^3}} \qquad (1)$$

where G is the gravitational constant, M is the mass of the asteroid (4.2 x 10^{16} kg), and a_0 is the initial distance from the origin to the satellite. Note that ω_{orb} is really a first order approximation; we are only considering the monopole term in the potential. Also note that in all cases the initial orbit is circular (to a first order approximation), thus no attempt has been made to include eccentricity in our parameter space.

In addition to these two variable parameters, three different orbit types were studied. These orbits are referred to as prograde, retrograde, and polar. Recalling that the spin axis is aligned with the z-axis, the following are the initial conditions for each case:

$$\text{Prograde:} \quad \mathbf{r} = (0, a_0, 0) \quad \mathbf{v} = \left(-\sqrt{\tfrac{G\ M}{a_0}}, 0, 0\right)$$

$$\text{Retrograde:} \quad \mathbf{r} = (0, a_0, 0) \quad \mathbf{v} = \left(\sqrt{\tfrac{G\ M}{a_0}}, 0, 0\right) \qquad (2)$$

$$\text{Polar:} \quad \mathbf{r} = (0, 0, a_0) \quad \mathbf{v} = \left(\sqrt{\tfrac{G\ M}{a_0}}, 0, 0\right)$$

Results

We are now in a position to test the stability of these orbits. The fairly arbitrary ranges of $\omega_{spin} \in [0.00, 6.00] \times 10^{-4} s^{-1}$ and $\omega_{orb} \in [0.32, 4.00] \times 10^{-4} s^{-1}$ were tested; note that this puts a_0 from a distance of ~ 140 km to within the Brillouin sphere of Ida. The results for retrograde, prograde, and polar are displayed in Figures (2,a), (b) and (c) respectively. Each of these figures is a 35 x 35 grid of orbits (that is, a total of 1225 1000-year orbits for each type of orbit). Black represents stable orbits while white represents unstable orbits. Note that an 8-year simulation was also performed and no significant differences were observed between these two time scales. Similar stability plots have been produced (although here we have defined our parameters quite differently) by Hu & Scheeres (2004). To the author's knowledge, however, this is the first time it has been done using an actual asteroid model.

The first thing one should note is that while Figures (2,b) and (c) show similar structure to each other, Figure (2,a) is drastically different; the orbital stability appears to depend primarily on ω_{orb}, with the critical value being in the neighbourhood of 2.7 x 10^{-4} s^{-1} (corresponding to a critical a_0 of about 34 km). The stability of retrograde orbits are, for the most part, independent of ω_{spin}. This result is quite surprising considering that the long axis of Ida extends to about 30 km from the origin; it shows that the satellite can be orbiting very closely to Ida and still be stable. It has long been thought that retrograde orbits are more stable than prograde, and the results obtained here give further support towards this conjecture.

It is important to keep in mind that the resolution in Figures (2,a), (b) and (c) is not very fine, and as such there is no observable fine structure. The coarse resolution is due to computational time constraints. It may be of interest, then, to zoom into the plots, particularly at the stability-instability boundaries. Figure (3) shows the result of zooming in on a section of the retrograde plot (using the 8-year results). Each successive zoom has, again, a 35 x 35 resolution. By doing this we can see that there is indeed a high degree of fine structure. The stability-instability boundary appears to be the same no matter how closely one zooms in on it; the boundary will still be jagged and uneven. That is, the plot appears to resemble a fractal! This applies to Figures (2,b) and (c) as well.

The most prominent feature of Figures (2,b) and (c) is the band of stability present for low ω_{spin}. Apparently, for small (but non-zero) values of ω_{spin}, the orbit is stable even when the satellite is well within the Brillouin sphere of Ida! This result is entirely unexpected, and as such special attention is paid to it. To study this type of orbit, we view the orbit in the body fixed frame of the asteroid (rather than the inertial frame). The polar orbit with parameters ω_{spin}= 0.1 765 x $10^{-4}s^{-1}$ and ω_{orb}=3.573 x $10^{-4}s^{-1}$ (corresponding to a_0 = z(t = 0) ≅ 28 km) was studied since it is both stable and within the Brillouin sphere of Ida. A plot was produced in which an (x,y) point was recorded every time the satellite passed through the x-y plane. Figure (4) shows the result of performing this procedure. Recalling that the x-axis corresponds to the long axis of Ida and the y-axis corresponds to the shorter, it can be seen that the orbit only reaches its pericenter (which is in the Brillouin sphere) when it is away from the long axis, and thus there seems to be a protection mechanism for keeping the satellite from crashing! This type of self-preserving orbit has been seen before (Petit *et al.* 1997), but not – to the author's best knowledge – in polar orbits.

Conclusion

The stabilities of satellite orbits around asteroid 243 Ida were numerically determined for a large range of orbital parameters.

Results support previous conclusions that retrograde orbits are, in general, more stable than prograde or polar orbits. In addition, prograde and polar orbits were both observed to have stability regions for slow (but non-zero) asteroid rotation, even when the orbit passes within the Brillouin sphere of Ida. This is seen to be the result of a self-preservation process by which the orbit only reaches its pericenter when it is away from the long axis of the asteroid. These results may prove useful in determining stable orbits for future visits to asteroids by spacecraft. Aside from application, these results are theoretically interesting as they are somewhat unexpected. An analytical analysis of the self-preservation phenomenon (particularly in orbits around Ida) would likely prove useful and enlightening.

Stability-instability boundaries in parameter space were found to be highly fine structured and fractal-like in nature. This is certainly not the first time that a fractal structure has been found to arise from natural phenomena. As a result, the observations made have great theoretical merit and much further study is warranted.

Further numerical studies of this system should look at a wider range of orbital parameters. Including orbital eccentricity as a variable parameter would also be a good place to continue research into this system.

Acknowledgements

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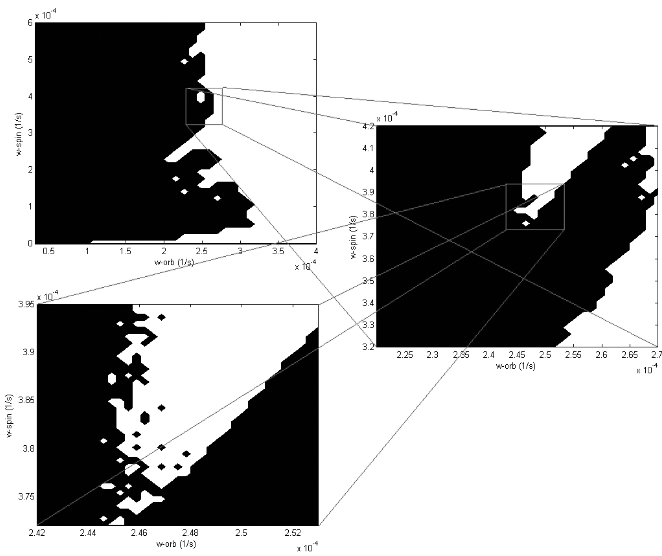

Figure 3: Shows that the stability plots contain a high degree of fine structure not visible using a 35x35 resolution. The stability-instability boundary appears to be fractal-like.

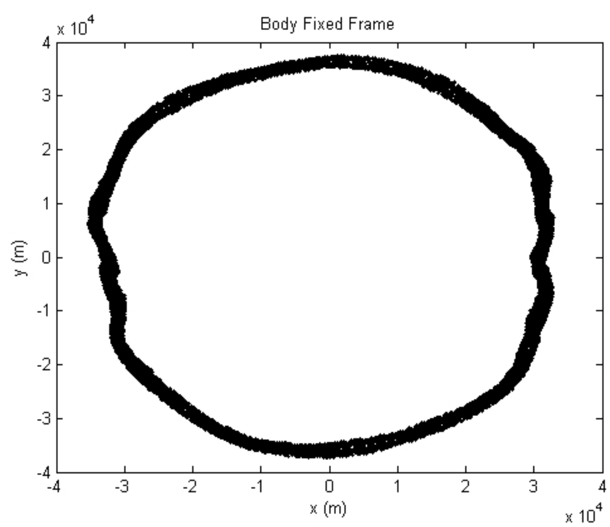

Figure 4: Displays the x-y cross-section (in the body fixed frame) of the polar orbit with parameters ω_{spin}=0.1765×$10^{-4}s^{-1}$ and ω_{orb}=3.573×$10^{-4}s^{-1}$. It can be seen that the satellite only reaches pericenter when it is away from the long axis (the x-axis in this case), and so is protected from crashing.