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The Stefan-Boltzman Law, the Wien Displacement Law and the temperature of the Sun

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Abstract

In this paper a straightforward approach involving logarithms is used to derive the Stefan-Boltzmann and Wien Displacement laws from measured data of light output. In the final portion of this paper, the Wien Displacement law is used to make an estimate of the surface temperature of the sun using data collected with a quartz prism and a CdS light meter.

Power law

Several power laws in physics were discovered before the underlying physics was fully understood. A number of these power laws are introduced in high school physics. For instance, Newton's Theory of Universal Gravitation is an inverse squared power law.

Two other important power laws are the Stefan-Boltzmann law and the Wien Displacement Law. The former describes the relationship between the radiant thermal emission of an object and its temperature, while the latter describes the relationship between an object's temperature and peak wavelength of radiant thermal emission.

From a practical standpoint we can relate colour to temperature. When we look at a candle, the flame is hottest near the wick and coolest near the outside of the flame. The wick is where the wax is combusting. Looking closely at the colour of the flame on a candle we see that the area nearest the wick is blue in colour, while the outside of the flame is orange or red. The colour of the flame is also related to the rate of cooling of the flame, which in turn is related to the temperature of the surrounding volume.

Precise measurements of the rate of cooling of a body as a function of temperature were first made in 1817 by the French scientists Dulong and Petit and were confirmed by the English scientist Tyndall in 1865. It was Austrian scientist Josef Stefan who first showed in 1879 that the rate of cooling was proportional to the fourth power of temperature. This was later justified theoretically by the Austrian physicist Ludwig Boltzmann in 1884.

Using logarithms to find a power law relationship

Logarithms are introduced in middle school mathematics. It is possible to derive power laws from measured data using logarithms.

An example of a power law is the fact that the area of a circle is related to the square of its radius. Let us for a moment assume we do not know this relationship and use logarithms to determine it. Consider

$$A_{circle} = k\,R^{\,n} \qquad (1)$$

where Acircle is the area of the circle, k is some unknown constant, R the radius measured in arbitrary units, and n is some yet to be determined power. It is straightforward to draw circles of differing radii on graph paper and then measure the area. A table for the area of a circle as a function of its radius is given in Table 1.

We can use logarithms to find the value of n. Consider that

$$A_2 / A_1 = (R_2 / R_1)^n \qquad (2)$$

then taking the natural logarithm (ln) of both sides of this equation and solving for n

$$n = \ln (A_2 / A_1) / \ln (R_2 / R_1) \qquad (3)$$

Using the data from table

$$n = \ln(153.4 / 12.6) / \ln(7.0 / 2.0) = 2.50 / 1.25 = 2 \qquad (4)$$

In turn, using the data we can also estimate k, that is k = 3.14 so then to the accuracy of the collected data, the area of a circle is given approximately by

$$A_{circle} = 3.14\,(R)^2 \qquad (5)$$

Area	radius (arbitrary units)
12.6	2.0
78.5	5.0
153.4	7.0

Table 1 Area of a Circle as a function of radius

If we make the circle area larger and the units of measurement more accurate we can find that the constant is around 3.1416. Of course, we define the number pi as the ratio of the area of a circle to the square of its radius. We leave it to the philosophers to reconcile the definition of a transcendental number through a ratio of two finite numbers.

The spectrum of radiant emission of a luminous body

We shall use the logarithmic technique to derive the Stefan- Boltzmann Law and the Wien Displacement Law. An object that is warmer than its surroundings will radiate energy. The luminous colour of an object is related to the temperature of the object.

A bolometer, an instrument that can precisely measure the total radiant emission of a heated object, can be used to measure the thermal spectrum of a heated object as a function of its temperature.

If the light is passed through a quartz prism, and the amount of energy is measured as a function of wavelength, the spectrum of thermal emission is obtained for the heated object. A prism made from quartz needs to be used and because normal glass absorbs infrared light.

The area under the curves of the different thermal spectra (figure 1) appears to be related in some fashion to the absolute temperature (T) of the spectra. The area under each curve (A_{curve}) is a measure of the total amount of the radiant energy emitted (U_{rad}) by the object at the specified temperature.

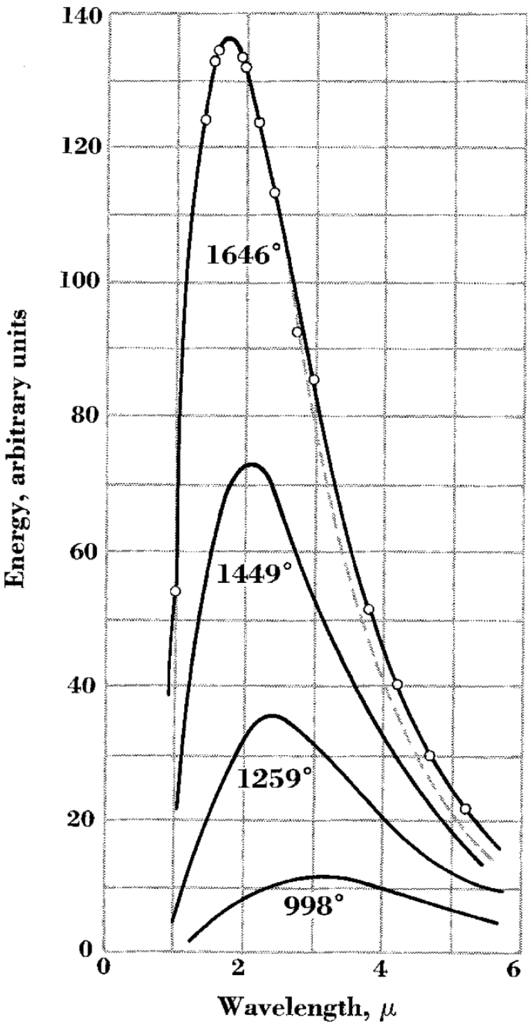

Figure 1 Thermal spectrum of an object at different temperatures

The wavelength of the peak of each curve also appears to be related to the absolute temperature. It appears that the hotter the object the shorter the wavelength of the peak of its spectrum.

Let us try to find a mathematical function that will relate the total energy emitted to the temperature of the object and the peak of the spectra to the temperature of the object. The total radiant energy as a function of temperature is given in Table 2.

Let us see if a power law can describe the functional relationship between the total energy emitted by the object as a function of temperature.

$$U_{rad} = \text{Constant } T^n \qquad (6)$$

where n is some yet to be determined power relationship. We will use logarithms to find the value of n. Taking the ratio of the total energy emitted at two temperatures we find

$$U_2 / U_1 = (T_2 / T_1)^n \qquad (7)$$

Taking the natural logarithm (ln) of both sides of this equation and solving for n

$$n = \ln (U_2 / U_1) / \ln (T_2 / T_1) \qquad (8)$$

From the data in table 3, and using the values for T = 998 K and T = 1646 K,

$$n = \ln (33.4 / 4.5) / \ln (1646 / 998) = 2.00 / .500 = 4.0 \qquad (9)$$

The Stefan-Boltzmann Law is then

$$U_{rad} = \text{Constant } T^4 \qquad (10)$$

The constant, known as the Stefan-Boltzman constant, is equal to 5.67 x 10^{-8} W /m^2 T^4. Using quantum mechanics it is possible to derive the Stefan_Boltzman law from first principles using Planck quantum theory of black body radiation.

The Wien Displacement Law

In 1893 the German physicist and Nobel Prize laureate Wilhelm Wien (1864-1928) discovered the relationship between the peak wavelength and the temperature of a luminous object.

Studying the spectra in Figure 1 we notice as that as the object became hotter the peak wavelength of its spectra became shorter. Table 3 lists the peak wavelength of the spectra in micrometers as a function of temperature.

Temperature (K)	Total emission (arbitrary units)
998	4.5
1259	12.0
1449	21.5
1646	33.4

Table 2 The total radiant energy as a function of absolute temperature

You can once again use the logarithm technique to find a relationship between peak λ_{peak} wavelength and temperature in Kelvin. The relationship, which is known as the Wien Displacement Law, is given by

$$\lambda_{peak}T = \text{constant} \qquad (11)$$

where the constant is 2.898 x 10^{-3} m-K.

The Wien Displacement Law allows physicists and astronomers to use the peak wavelength of a luminous object to measure its surface temperature. It is used extensively in astronomy.

The temperature of the surface of the sun

Using a quartz prism and a CdS light meter the following readings were made of the luminous output of the sun (table 4). From this table, we can see that the peak wavelength of the solar output is yellow, about 0.500 μm, which means that the surface temperature of the sun is

$$T = 2.898 \times 10^{-3}\ \text{m-K} / 5.0 \times 10^{-7}\ \text{m} = 5796\ \text{K} \qquad (12)$$

The Wien Displacement law is used extensively in astronomy to study celestial objects far removed from earth, objects like the planets (whose atmospheric temperature can be measured remotely using bolometers or CCD's), stars many light years away, beautiful nebulae, whole galaxies millions of light years away and even the universe itself. The Cosmic Microwave Background temperature, which is 13.7 billion light years away, is measured at around 2.7 K using the Wien Displacement law. The fluctuation in the CMB, measured in milliKelvin has also been measured and mapped by satellites and forms the basis of modern theories of the birth and evolution of the universe.

Canadians and the Cosmic Microwave Background

One of the first astronomers to measure the background temperature of interstellar space was the Canadian Dr. Andrew McKellar (1910-1950). In 1940 Dr. McKellar was the first astronomer to detect the presence of carbon compounds in interstellar space, and in 1941 he measured a temperature of approximately 2.5 K in these organic cyanogens compounds using the Dominion Astrophysical Observatory telescope in Victoria.

Dr. McKellar made his discoveries some twenty five years before Arno Penzias and Robert Wilson confirmed these measurements in 1964 and went on to win a Nobel Prize for detecting the "Cosmic Microwave Background". While Penzias and Wilson unintentionally measured the cosmic background radiation while conducting diagnostic observations using a Bell Laboratory microwave receiver built for the ECHO satellite experiments, McKellar had intentionally set out to measure the ambient temperature of interstellar clouds and therefore the ambient temperature of the universe.

Temperature (K)	Peak Wavelength λ_{peak} in μm
998	2.98
1259	2.35
1449	2.05
1646	1.80

Table 3 Peak Wavelength in micrometers as a function of absolute temperature

No mention of Dr. McKellar's work is to be found in either of the Nobel Prize addresses by Penzias and Wilson, a glaring oversight on their part. Unfortunately, the Nobel Prize Committee does not award prizes posthumously. Dr. McKellar died in 1960 at the young age of fifty some four years before the 1964 microwave radiometric measurements and some eighteen years before Penzias and Wilson won their Nobel Prize in 1978 1. In 1962 the Dominion Astrophysical Observatory in Victoria named their 1.2 metre refractor telescope in honour of Dr. Andrew McKellar.

There is a rather deep feeling amongst Canadian astrophysicists, astronomers and physicists that Canadian born Dr. Jim Peebles should have received one-quarter of the 1978 Nobel Prize in Physics. Born in Winnipeg in 1935, Dr. Peebles completed his bachelor's degree at the University of Manitoba before going onto his doctorate at Princeton. Dr. Peebles made many important contributions to the Big Bang model and wrote extensively on the theoretical underpinnings to the cosmic microwave background radiation with Robert Dicke.

Ceteris paribus, in the opinion of one of the authors Penzias and Wilson were less deserving than other candidates for a Nobel Prize on the Big Bang and the Cosmic Microwave Background.

References

1. See http: // astro-canada.ca/_en/a2207.html

Natalia Kalicki and **Tatyana Olal** are students at a private middle school in Vancouver. They have helped to write this paper as a community service project in support of the 2009 International Year of Astronomy. Natalia is interested in pursuing a career in the fine arts and Tatyana is interested in literature and journalism. Both students are happy to be part of the 2009 IYA celebrations in Canada.

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Colour	Reading (arbitrary units)
Infrared	15
Red	45
Orange	70
Yellow	75
Green	48
Blue	30
Violet	10

Table 4 Photometer reading as a function of colour